Reliable Method for Computing the Phase Shift of Multiline LRL Calibration Technique

J. A. Reynoso-Hernández

***Abstract*—In this paper, a new method for computing the phase shift of lines used in the multiline line–reflect–line (m) calibration technique is presented. The new method is based on a matrix approach to determine the traveling wave λ and uses a reliable criterion to compute the constants a/c and b. The new method overcomes the problem of sign ambiguity of the eigenvalues λ_1 and λ_2, inherent to the classical method. Another advantage of the new method of computing the phase shift is that the physical length of lines is not needed. The new method computes the line phase shift from the knowledge of the traveling wave λ, that is, without previous knowledge of the wave propagation constant.**

***Index Terms*—Calibration techniques, phase shift, wave cascade matrix, wave propagation constant.**

I. Introduction

THE MULTILINE line–reflect–line (LRL) (m) calibration technique utilizes many different combinations of pairs of lines to avoid the phase shift uncertainty regions located at $180°k$ ($k = 0, 1, 2, \ldots, n$). Usually, the phase shift of a pair of lines used in the calibration procedure is determined from the previous knowledge of the imaginary part of the wave propagation constant γ [1]. Either the ABCD matrix or the wave cascading matrix (WCM) formalisms may be used for γ computation. In fact, utilizing the similar matrix properties and either the ABCD or the WCM, the first step in the broad-band γ calculation is the computation of the traveling wave λ [1]. To date, the main problem with the broad-band λ computation when using either ABCD or WCM is the lack of a reliable criterion to resolve the sign ambiguity between the two eigenvalues λ_1 and λ_2 [1], [2], predicting a continuous forward or backward traveling wave. On the other hand, the method using the wave cascade matrix seems to be more appropriate and compatible with the calibration procedure. Irrespective of the parameters used for the broad-band γ computation, the method L–L requires the physical dimensions of the lines as input data. The drawback of these methods [1], [2] is that it is necessary to know the physical length of the lines to compute γ. Based on the matrix development presented by [3], a new and reliable method for determining the phase shift of a pair of lines used in the LRL(m) is presented in this paper. The main feature of this new method is that the physical lengths of lines are not needed and by consequence the γ computation and the sign ambiguity are avoided.

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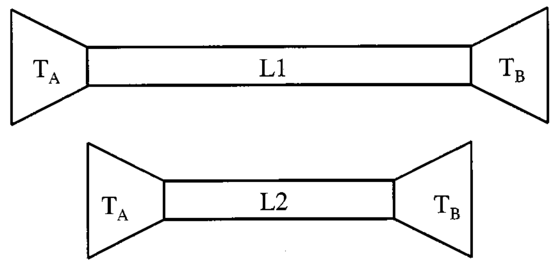

Fig. 1. Structures used for the implementation of the line–line method.

II. Method

The method needs for its implementation two nonreflecting lines. The line standards used are shown in Fig. 1. The shorter and longer lines will be referred to as L_1, and L_2, respectively. The two ports that are referenced to as T_A and T_B correspond to transitions used for ensuring the connection between the lines and the network analyzer at the line input and output ports. T_A and T_B include either the microwave probes or coaxial to microstrip microwave connectors (launchers) and the necessary hardware for the network analyzer.

WCMs are used for the modeling of transitions T_A, T_B, line L_1 and line L_2. The WCM is defined as $\mathbf{T_A}$ for transition T_A, $\mathbf{T_B}$ for transition T_B, $\mathbf{T_{L1}}$ for line L_1, and $\mathbf{T_{L2}}$ for line L_2. In the following, $\mathbf{T_A}$ and $\mathbf{T_B}$ are assumed to be different. The WCM $\mathbf{T_1}$, and $\mathbf{T_2}$ ($\mathbf{T_1}$ for line L_1 plus transitions, $\mathbf{T_2}$ for line L_2 plus transitions) can be written as

$$\mathbf{T_1} = \mathbf{T_A}\,\mathbf{T_{L1}}\,\mathbf{T_B} \tag{1}$$

$$\mathbf{T_2} = \mathbf{T_A}\,\mathbf{T_{L2}}\,\mathbf{T_B} \tag{2}$$

where

$$\mathbf{T_{Li}} = \begin{pmatrix} e^{-\gamma Li} & 0 \\ 0 & e^{\gamma Li} \end{pmatrix}, \qquad i = 1, 2 \tag{3}$$

$$\mathbf{T_A} = r_{22}\begin{pmatrix} a & b \\ c & 1 \end{pmatrix}. \tag{4}$$

$\mathbf{T_{Li}}$ ($i = 1, 2$) is the WCM of a nonreflective line having a length L_i.

Combining (1) and (2) the next equation results

$$T_\lambda = T_A^{-1} T T_A \tag{5}$$

where

$$T_\lambda = T_{L2}T_{L1}^{-1} = \begin{pmatrix} \dfrac{1}{\lambda} & 0 \\ 0 & \lambda \end{pmatrix}, \qquad \lambda \in C, \tag{6}$$

$$T = T_2 T_1^{-1} = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix}, \qquad t_{ij} \in C \tag{7}$$

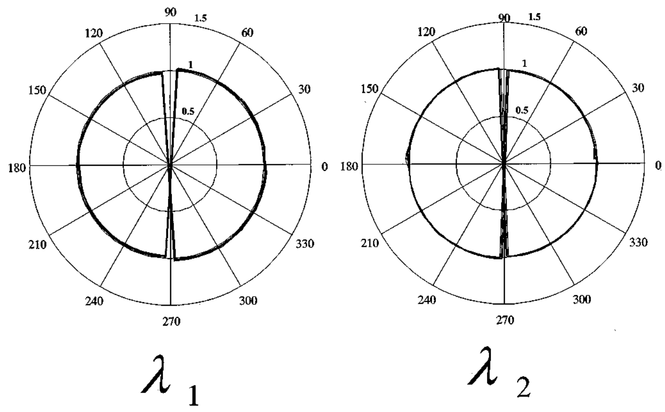

Fig. 2. Polar plots of the eigenvalues λ_1 and λ_2 illustrating a discontinuous behavior in phase and magnitude around 90° and 270°.

and

$$\lambda = e^{\gamma(L_2 - L_1)}. \tag{8}$$

Using (4) and (7), (5) becomes (9), shown at the bottom of the page.

Comparing each term of matrices on both sides of (9), λ is computed as a function of b and a/c by

$$\begin{aligned}\lambda &= \frac{t_{22} + bt_{21} - \frac{b}{a/c} t_{11} - \frac{1}{a/c} t_{12}}{1 - \frac{b}{a/c}} \\ &= \frac{1 - \frac{b}{a/c}}{t_{11} + \frac{1}{a/c} t_{12} - bt_{21} - \frac{b}{a/c} t_{22}}\end{aligned} \tag{10}$$

where

$$b^2 t_{21} + b(t_{22} - t_{11}) - t_{12} = 0 \tag{11}$$

$$\left(\frac{a}{c}\right)^2 t_{21} + \frac{a}{c}(t_{22} - t_{11}) - t_{12} = 0. \tag{12}$$

It should be noted that the quadratic equations (11) and (12) have already been reported in [4], but their derivation is different [3]. On the other hand, because of the equal coefficients in (11) and (12), b and a/c are roots of the same equation. Moreover, since T_A^{-1} exists $(a - bc \neq 0)$, then b is different from a/c and, hence, b and a/c are the two different roots of (11) and (12). Values of b and a/c are chosen in accordance to the criterion reported in [4].

It is interesting to comment that the λ expression given by equation (10) predicts a continuous wave traveling in positive direction [3]. This is an improvement in the λ calculation since the problem of sign ambiguity inherent to methods [1], [2] does not exist in the proposed method. Once λ is computed the phase shift is determined by

$$\Delta\phi = \arctan\left(\frac{\mathrm{Im}(\lambda)}{\mathrm{Re}(\lambda)}\right). \tag{13}$$

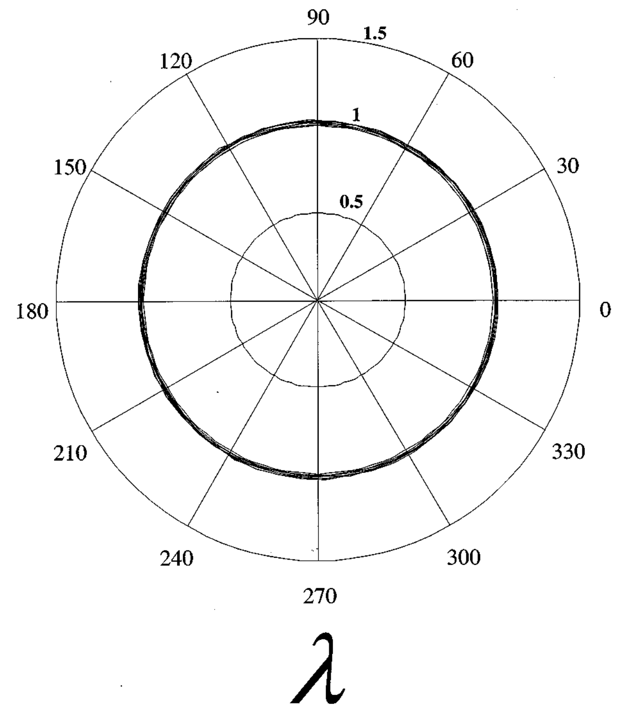

Fig. 3. Polar plot of the traveling wave λ illustrating a continuous behavior in phase and magnitude in broadband.

III. Experimental Results

Two precision air lines (7943G and 7943H from Maury Microwave) and an uncalibrated HP8510C automatic network analyzer system were utilized to test the reliability of the method for computing the phase shift of lines used in multiline calibration technique. The measurements of the scattering parameters of the lines were performed in the frequency range of 0.045–50 GHz with an uncalibrated HP8510C system. Fig. 2 shows the broad-band variation of λ_1 and λ_2 versus frequency, in the complex plane, computed using [1, eq. (20)]. The discontinuous behavior in phase and magnitude of λ_1 and λ_2 observed in the vicinity of 90° and 270° precludes the direct broad-band determination of the phase shift of lines using λ_1 or λ_2. Fig. 3 shows the broad-band variation versus frequency of λ, in the complex plane, determined using (10). It should be noted that a continuous phase and magnitude variation in the whole frequency band is observed in Fig. 3. The continuous phase variation of λ enables the broad-band determination of the phase shift of lines without previous knowledge of the wave propagation constant. Finally, the effective phase shift of the lines [5], determined using (13), is depicted in Fig. 4. It should be noted that the variations of the effective phase shift as a function of the frequency produce a periodic curve for a given pair of lines. Furthermore, it is interesting to observe that the

$$\begin{pmatrix} \frac{1}{\lambda} & 0 \\ 0 & \lambda \end{pmatrix} = \frac{1}{a - bc} \begin{pmatrix} at_{11} + ct_{12} - abt_{21} - bct_{22} & bt_{11} + t_{12} - b^2 t_{21} - bt_{22} \\ -act_{11} - c^2 t_{12} + a^2 t_{21} + act_{22} & -bct_{11} - ct_{12} + abt_{21} + at_{22} \end{pmatrix} \tag{9}$$

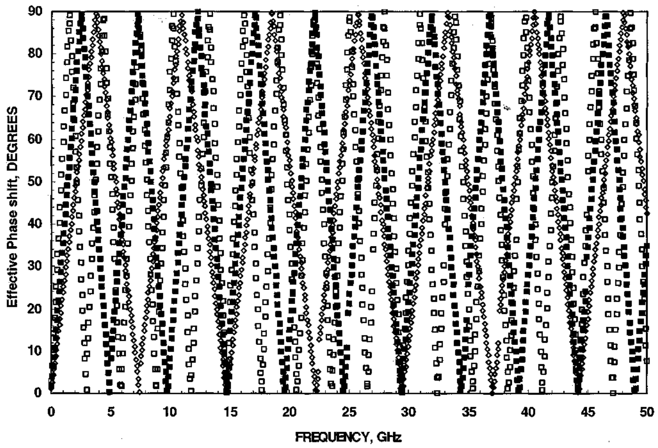

Fig. 4. Effective phase shift of the different pair of lines combination. The fill squares represent the Thru-Line1 (4.997 cm), the open squares represent the Thru-line2 (2.998 cm), and the open diamonds represent the difference between Line1–Line2.

effective phase shift of lines depicted in Fig. 4 approaches 0° and 90° at some specific frequencies. In fact, the pair of lines cannot be used at frequencies where the effective phase shift reaches 0° because the constants $(b,\, a/c)$ characterizing the network analyzer become ill-conditioned.

IV. Conclusion

Using a new matrix approach, a reliable method for computing the phase shift of the lines, used in the multiline LRL calibration technique, was developed. While the classical method uses the line wave propagation constant and the physical length of the lines for computing the phase shift, the proposed method uses a new matrix approach for computing λ along with a reliable criterion to compute the constants $\boldsymbol{a/c}$ and $\boldsymbol{b}$. Thanks to the novel matrix approach, the sign ambiguity problem for determining a continuous traveling-wave vector λ inherent in the classical method is avoided.

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